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Evading non-Gaussianity consistency in single field inflation

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1. Introduction

non-Gaussianity consistency relation

For "any" single-field model

$$\left\langle \mathcal{R}_{k_1}\mathcal{R}_{k_2}\mathcal{R}_{k_3}\right\rangle = (2\pi)^3\delta^3(k_1+k_2+k_3)(1-n_s)P(k_1)P(k_2)$$

for $k_1 \ll k_2 \approx k_3$ (squeezed limit ≈ local f_{NL})

$\mathcal{R}$: comoving curvature perturbation

$$\frac{12}{5}f_{NL} = 1-n_s = 2\varepsilon+\eta;\ \varepsilon \equiv -\frac{\dot{H}}{H^2},\ \eta \equiv \frac{\dot{\varepsilon}}{H\varepsilon}$$

Creminelli & Zaldarriaga ('04), Chen et al. ('06), Ganc & Komatsu ('10)

But this "theorem" assumes
constancy of $\mathcal{R}$ on superhorizon scales

2. $\mathcal{R}$ on superhorizon scales

$$S=\frac{1}{2}\int d^3x\,d\tau\,z^2\left[\mathcal{R}'^2-c_s^2(\nabla\mathcal{R})^2\right] \quad ; \; z^2\equiv\frac{2\varepsilon a^2 M_P^2}{c_s^2}$$

$$\Rightarrow \quad \left[\frac{1}{z^2}\frac{d}{d\tau}\left(z^2\frac{d}{d\tau}\right)+c_s^2k^2\right]\mathcal{R}=0; \quad d\tau=\frac{dt}{a}, \quad {}'=\frac{d}{d\tau}$$

- if $z'/z=O(a'/a=\mathcal{H})$

  general solution on super (sound-)horizon scales is

$$\mathcal{R}_k=C_k+D_k\int^\tau\frac{d\tau}{z^2} \qquad \Leftarrow \left(c_s k\ll\left|\frac{z'}{z}\right|=O(\mathcal{H})\right)$$

- conventionally D_k-term is decaying because $z\propto a$

  $(\varepsilon/c_s^2\approx const.\ll 1)$

growing vs decaying

$$\mathcal{R}_k = C_k + D_k \int_0^\tau \frac{d\tau}{z^2}$$

What if D_k term is not decaying?

- For simplicity, consider the case c_s=1 $\Rightarrow$ $z^2 = 2\varepsilon a^2 M_P^2$

If ε decays faster than a^{-3}, D_k term grows!

Let $\varepsilon \propto a^{-2n}$ $\left(\varepsilon = -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2H^2} \propto a^{-2n} \right)$

$$\Rightarrow \quad \varepsilon a^2 \propto a^{2-2n} \propto (-\tau)^{2n-2}$$

n~0 for standard slow-roll

- For $n < \frac{3}{2}$, D_k-term vanishes as $\tau \to 0$; C_k dominates

- For $n > \frac{3}{2}$, D_k-term grows indefinitely as $\tau \to 0$

scalar perturbation on flat slices

$$\begin{cases} \delta\phi_F = \delta\phi \text{ on flat slices} \\ \mathcal{R} = \text{ curvature perturbation on } \delta\phi = 0 \text{ slices} \end{cases}$$

$$\delta\phi_F = -\frac{\dot{\phi}}{H}\mathcal{R} \propto \sqrt{\varepsilon}\mathcal{R} \propto a^{-n}\mathcal{R} \qquad \left(a \sim (-H\tau)^{-1} \text{ for de Sitter}\right)$$

$$\Rightarrow \begin{cases} \mathcal{R} = C + D\int^{\tau} \frac{d\tau}{\varepsilon a^2} \sim C + D\int^{\tau} \frac{d\tau}{a^{2-2n}} \sim C + D a^{2n-3} \\ \delta\phi_F \propto a^{-n}\left(C + D a^{2n-3}\right) = C a^{-n} + D a^{n-3} \end{cases}$$

Both for n=0 and 3, there is a constant mode in $\delta\phi_F$

but the roles of C and D terms are interchanged

scale-invariant spectrum not only for n=0 but also for n=3

$\varepsilon \propto a^0$ $\qquad$ $\varepsilon \propto a^{-6}$

4. Simple toy model

constant potential: $V=V_0$

$$\ddot{\phi}+3H\dot{\phi}=0 \quad \Rightarrow \quad \dot{\phi}\propto a^{-3}$$

$$H^2=\frac{1}{3M_P^2}\left(\frac{1}{2}\dot{\phi}^2+V_0\right)\approx\frac{V_0}{3M_P^2}$$

$$\varepsilon\propto\dot{\phi}^2\propto\frac{1}{a^6} \quad \Rightarrow \quad \varepsilon=\varepsilon_e e^{6N}$$

"ultra slow-roll" (USR) inflation

Kinney '05

$$\eta=\frac{\dot{\varepsilon}}{H\varepsilon}=-6$$

N : # of e-folds from the end of inflation

$$a=a_e e^{-N}$$

necessary to recover scale-invariant spectrum

scalar field perturbation

- action for $\delta\phi_F$

$$S = \int d^3x\, d\tau \frac{a^2}{2}\left[\delta\phi_F'^2 - (\nabla\delta\phi_F)^2 - m_{eff}^2\delta\phi_F^2\right]$$

$$m_{eff}^2 = \partial_\phi^2 V + \frac{2}{M_P^2}\frac{d}{dt}\left(\frac{V}{H}\right) = 2\varepsilon\frac{V}{M_P^2} \approx 0$$

~ pure massless minimally coupled field on pure dS

- quantization

$$\delta\phi_F = \int \frac{d^3k}{(2\pi)^{3/2}}\left(a_k\phi_k(t)e^{ikx} + h.c.\right)$$

$$\Rightarrow \quad \phi_k = \frac{H}{(2k)^{3/2}}(1+ik\tau)e^{-ik\tau} \rightarrow \frac{H}{(2k)^{3/2}}$$

ϕ_k freezes on superhorizon scales as usual

5. Extended δN formula

Let us assume that USR inflation ends at $\phi = \phi_e$

Integrating eq of motion, $\ddot{\phi} + 3H\dot{\phi} = 0$:

$$\dot{\phi} = \frac{M^2}{3H} e^{-3Ht}, \qquad \phi = \frac{M^2}{3H}\left(e^{-3Ht_e} - e^{-3Ht}\right) + \phi_e$$

$$\Rightarrow \quad \phi(t) - \phi_e = \frac{\dot{\phi}(t)}{3H}\left(e^{-3N} - 1\right)$$

$$\Rightarrow \quad N = N(\phi, \dot{\phi}) = \frac{1}{3}\ln\left[\frac{\dot{\phi}}{\dot{\phi} + 3H(\phi - \phi_e)}\right]$$

N is a function of both ϕ and $\dot{\phi}$

curvature perturbation spectrum and non-Gaussianity

- as usual, one computes δN between the initial flat slice and the final comoving slice

- only difference is that $N=N(\phi, \dot{\phi})$ instead of $N=N(\phi)$

$$\delta N = \frac{\partial N}{\partial \phi}\delta\phi + \frac{\partial N}{\partial \dot{\phi}}\delta\dot{\phi} + \frac{1}{2}\frac{\partial^2 N}{\partial \phi^2}\delta\phi^2 + \frac{\partial^2 N}{\partial \phi \partial \dot{\phi}}\delta\phi\delta\dot{\phi} + \frac{1}{2}\frac{\partial^2 N}{\partial \dot{\phi}^2} + \cdots$$

$$= \frac{\partial N}{\partial \phi}\delta\phi + \frac{1}{2}\frac{\partial^2 N}{\partial \phi^2}\delta\phi^2 + \cdots$$

$$\delta\dot{\phi} \approx 0$$

$$= -\frac{H}{\dot{\phi} + 3H(\phi - \phi_e)}\delta\phi + \frac{3H^2}{2\left(\dot{\phi} + 3H(\phi - \phi_e)\right)^2}\delta\phi^2 + \cdots$$

- spectrum

$$\mathcal{P}_R = \left|\frac{H}{\dot\phi + 3H(\phi-\phi_e)}\right|^2 \left(\frac{k^3}{2\pi^2}|\phi_k|^2\right)_{k=\mathcal{H}} = \frac{H^2}{\dot\phi_e^2}\frac{H^2}{(2\pi)^2} = \frac{1}{2\varepsilon_e}\frac{H^2}{(2\pi M_P)^2}$$

scale-invariant spectrum: $n_s - 1 = 0$

- non-Gaussianity

$$\frac{3}{5}f_{NL} = \frac{1}{2}\frac{\partial_\phi^2 N}{(\partial_\phi N)^2} = \frac{3}{2}$$

apparently $\frac{3}{5}f_{NL} \neq 0 = \frac{1-n_s}{4}$

violation of non-Gaussianity consistency relation!

6. in-in formalism

3rd order action for comoving curvature perturbation

Maldacena '03

$$S_3 = \int dt d^3x \Big[a^3 \epsilon^2 \mathcal{R}\dot{\mathcal{R}}^2 + a\epsilon^2 \mathcal{R}(\partial\mathcal{R})^2 - 2a\epsilon\dot{\mathcal{R}}(\partial\mathcal{R})(\partial\chi) + \frac{a^3\epsilon}{2}\dot{\eta}\mathcal{R}^2\dot{\mathcal{R}} + \frac{\epsilon}{2a}(\partial\mathcal{R})(\partial\chi)\partial^2\chi + \frac{\epsilon}{4a}(\partial^2\mathcal{R})(\partial\chi)^2 + 2f(\mathcal{R})\frac{\delta L}{\delta\mathcal{R}}\Big|_1 \Big],$$

$$\partial^2\chi = a^2\epsilon\dot{\mathcal{R}}, \qquad \frac{\delta L}{\delta\mathcal{R}}\Big|_1 = a\left(\partial^2\dot{\chi} + H\partial^2\chi - \epsilon\partial^2\mathcal{R}\right)$$

$$\begin{aligned} f(\mathcal{R}) &= \frac{\eta}{4}\mathcal{R}^2 + \frac{1}{H}\mathcal{R}\dot{\mathcal{R}} \\ &+ \frac{1}{4a^2H^2}\left[-(\partial\mathcal{R})(\partial\mathcal{R}) + \partial^{-2}\left(\partial_i\partial_j(\partial_i\mathcal{R}\partial_j\mathcal{R})\right)\right] \\ &+ \frac{1}{2a^2H}\left[(\partial\mathcal{R})(\partial\chi) - \partial^{-2}\left(\partial_i\partial_j(\partial_i\mathcal{R}\partial_j\chi)\right)\right] . \end{aligned}$$

- Neglecting O(ε^2) and O(k^2) terms

$$S_3 = \int dt\, d^3x \left[\frac{a^3\varepsilon}{2} \dot{\eta} \mathcal{R}^2 \dot{\mathcal{R}} + (\varepsilon^2, k^2\text{-terms}) \right] + 2 f(\mathcal{R}) \left.\frac{\delta L}{\delta \mathcal{R}}\right|_1$$

$$f(\mathcal{R}) = \frac{\eta}{4}\mathcal{R}^2 + \frac{1}{H}\mathcal{R}\dot{\mathcal{R}} + (k^2\text{-terms})$$

- field redefinition:

$$\mathcal{R} \to \mathcal{R} = \mathcal{R}_n + f(\mathcal{R}_n) \approx \mathcal{R}_n + \frac{\eta}{4}\mathcal{R}_n^2 + \frac{1}{H}\mathcal{R}_n \dot{\mathcal{R}}_n$$

at USR stage

$$\dot{\mathcal{R}}_n = 3H\mathcal{R}_n \quad \to \quad \mathcal{R} \approx \mathcal{R}_n + \frac{\eta + 12}{4}\mathcal{R}_n^2 = \mathcal{R}_n + \frac{3}{2}\mathcal{R}_n^2$$

$\frac{3}{5} f_{NL} = \frac{3}{2}$ • • • in agreement with δN result

transition from USR to SR

If SR follows after USR, $\eta = \begin{cases} \eta_{USR} \approx -6 & \text{at USR stage} \\ \eta_{SR} \ll 1 & \text{at SR stage} \end{cases}$

f_{NL} from field redefinition is SR suppressed at SR stage:

$$\mathcal{R} \to \mathcal{R} = \mathcal{R}_n + f(\mathcal{R}_n) \approx \mathcal{R}_n + \frac{\eta}{4}\mathcal{R}_n^2 \approx \mathcal{R}_n + (\text{SR suppressed})$$

However, there appears a step in η :

$$\eta(t) \approx \eta_{USR}\left(1-\theta(t-t_c)\right) \ ; \quad \eta_{USR} \approx -6$$

$$\Rightarrow \quad \dot{\eta}(t) \approx -\eta_{USR}\delta(t-t_c)$$

$$\Rightarrow \quad S_3 \supset \int dt\, d^3x \left[\frac{a^3\varepsilon}{2}\dot{\eta}\mathcal{R}^2\dot{\mathcal{R}}\right] \approx \int d^3x \left[-\frac{a^3\varepsilon}{2}\eta_{USR}\mathcal{R}^2\dot{\mathcal{R}}\right](t_c)$$

This gives exactly the same f_{NL} obtained at USR stage

7. generalization

Chen, Firouzjahi, Namjoo & MS '13

- K-inflation type model

$$S=\int d^4x\sqrt{-g}P(X,\phi)\,;\ P=X+\frac{X^\alpha}{M^{4\alpha-4}}-V(\phi),\ X=-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

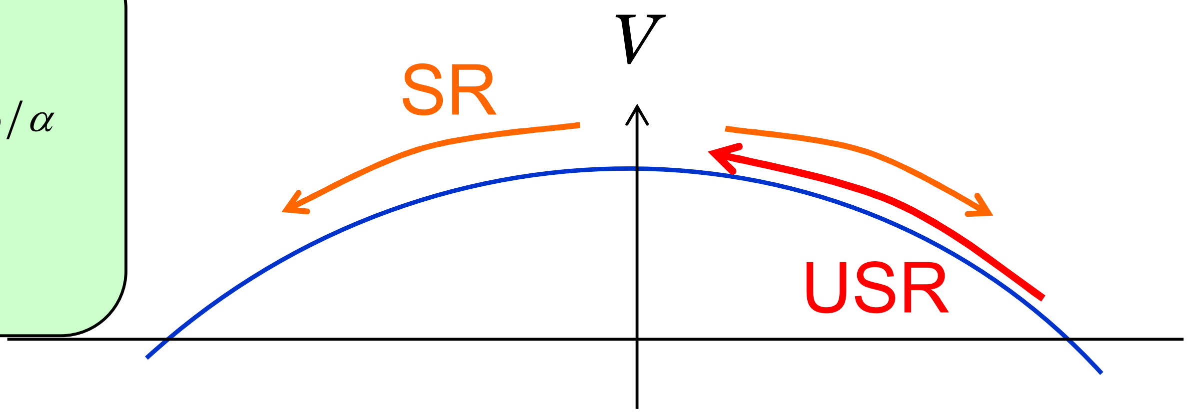
$$c_s^2=\frac{P_{,X}}{P_{,X}+2XP_{,XX}}=\frac{1+\alpha(X/M^4)^{\alpha-1}}{1+\alpha(2\alpha-1)(X/M^4)^{\alpha-1}}$$

$$\approx\frac{1}{2\alpha-1}\ll 1 \text{ for } \alpha\gg 1 \text{ and } X\gg M^4$$

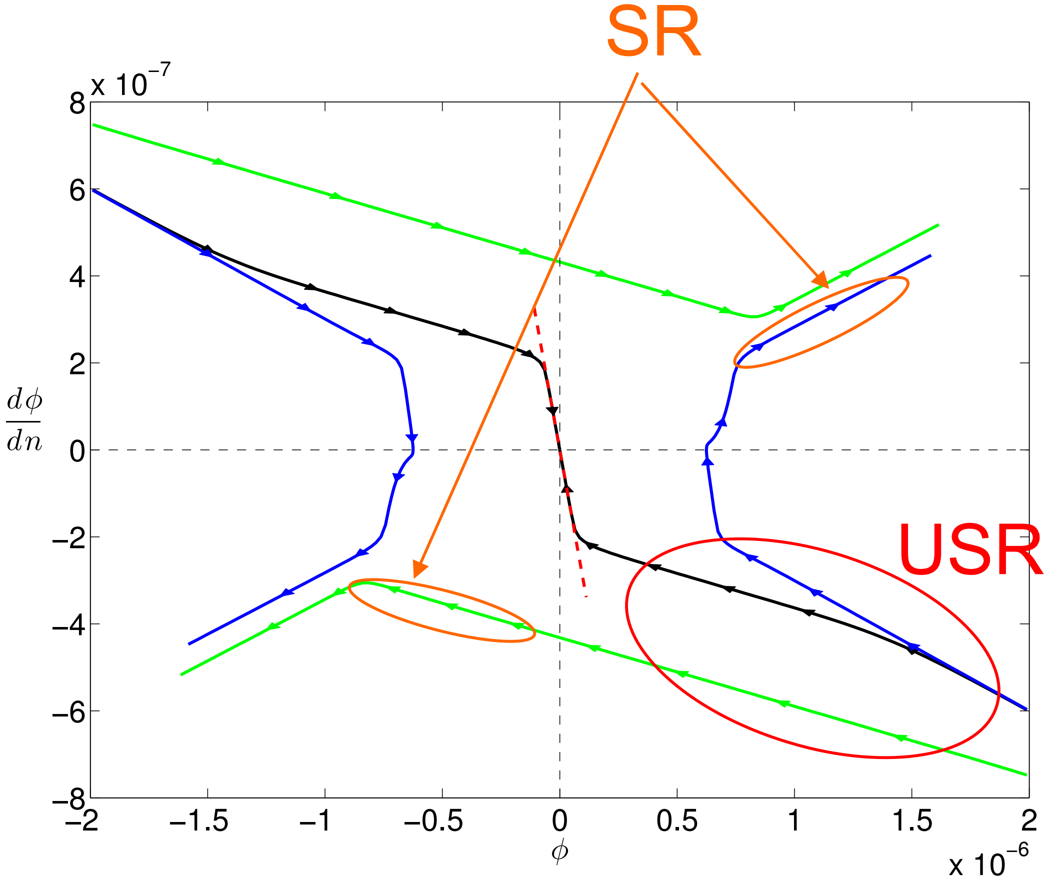
for $V\propto V_0-\upsilon\phi^{2\alpha}$,

$^\exists$a solution $X\propto a^{-6/\alpha}$

& $\eta\approx -6$ (USR)



phase diagram



In this case, one obtains almost scale-invariant spectrum, and

$$\frac{3}{5} f_{NL} = \frac{3}{4c_s^2}(1+c_s^2) \gg 1 \text{ for } c_s^2 \ll 1$$

large “local” f_{NL} is possible even in single-field inflation

8. Summary

- Non-Gaussianity consistency relation for single-field inflation,

$$\frac{12}{5} f_{NL} = 1 - n_s = 2\varepsilon + \eta; \ \varepsilon \equiv -\frac{\dot{H}}{H^2}, \ \eta \equiv \frac{\dot{\varepsilon}}{H\varepsilon}$$

can be violated if $\mathcal{R}$ grows on superhorizon scales

- Ultra slow-roll inflation can provide such a stage

- In particular, for X^{α} model with c_s<<1, f_{NL} can be very large.

NB. PLANCK 2013: f_{NL}<10

non-zero f_{NL} does not necessarily exclude all single-field models!